# Equilibrium Acid Concentrations in Hydrolyzed Folyesters and Polyester-Polyurethane Elastomers* 

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## Synopsis


#### Abstract

Equilibrium acid concentrations, $\left[\mathrm{A}_{e}\right]$, were determined in butylene adipate and caprolactone polyesters, of low molecular weight, and in thermoplastic polyurethane elastomers based on these polyesters. Values of [ $\mathrm{A}_{e}$ ] of the polyesters at $85^{\circ} \mathrm{C}$ were $0.7,1.3$, and $3-4 \times 10^{-3} \mathrm{~mol} / \mathrm{g}$ at relative humidities ( RH ) of $25 \%, 50 \%$, and $93 \%$, respectively. [ $\mathrm{A}_{e}$ ] of the thermoplastic elastomers at $85^{\circ} \mathrm{C}$ were about 3 and $7 \times 10^{-4} \mathrm{~mol} / \mathrm{g}$ at $10 \%$ and $25 \% \mathrm{RH}$, respectively. Values of $\left[\mathrm{A}_{e}\right]$ were not very dependent on temperature at constant RH. Equilibrated thermoplastic elastomers had low molecular weights and poor physical properties. Consequently, equilibration does not set a practical limit on hydrolytic degradation, even at low RH. Equations were developed that described the variation in acid content with time. Rate constants for hydrolysis and esterification increased as RH decreased. Reesterification in the elastomers in the absence of water is too slow to be a useful method of decreasing hydrolytic damage.


## INTRODUCTION

Cuddihy ${ }^{2}$ aged magnetic tapes at various temperatures and relative humidities (RH) in order to study the polyester-polyurethane binder that holds the magnetic particles on the tape base. The binder was highly crosslinked, so Cuddihy measured the fraction soluble in acetone. This fraction increased if tapes were aged at $100 \%$ and $30 \% \mathrm{RH}$ but decreased if tapes were aged at $11 \%$ and $0 \% \mathrm{RH}$. Increases in the soluble fraction were attributed to hydrolytic scission of ester groups, thus changing some gel to soluble material. Decreases in the soluble fraction were attributed to esterification reactions between acid and alcohol groups, which incorporated soluble molecules into the gel. Cuddihy concluded that the binder on his tapes should appear stable to hydrolysis at about $24 \% \mathrm{RH}$ at $20^{\circ} \mathrm{C}$ because of the dynamic equilibrium between hydrolysis and esterification. Very recently, Bertram and Cuddihy ${ }^{3}$ applied another kinetic analysis to Cuddihy's data ${ }^{2}$ and now suggest that tapes stored at $18^{\circ} \mathrm{C}$ and $40 \% \mathrm{RH}$ should appear stable to hydrolysis because of this equilibrium.

Equilibrations of ester and water with acid and alcohol have been studied before, ${ }^{4}$ but specific information is lacking at the lower water contents associated with environmental aging. The equilibration should occur in any polyester, whether it is crosslinked, not crosslinked, or incorporated into a polyurethane. Studies described herein used soluble polyesters of low molecular weight and polyester-polyurethane elastomers at several RH to find out if equilibration

[^0]limits degradation to acceptable values and if it is practical to diminish hydrolytic damage that has occurred by aging polyesters at low RH.

## KINETIC CONSIDERATIONS

Here we give preliminary kinetic considerations and derive equations consistent with them. The equilibrium between hydrolysis and esterification is described by the reactions.

$$
\begin{equation*}
\mathrm{E}+\mathrm{W} \underset{k^{\prime}}{\stackrel{k}{\leftrightarrows}} \mathrm{~A}+\mathrm{H} \tag{1}
\end{equation*}
$$

where $\mathrm{E}, \mathrm{W}, \mathrm{A}$, and H represent ester, water, acid, and alcohol, respectively. It is convenient to follow the equilibration by measuring the acid concentration at various times. Concentration units will be $\mathrm{mol} / \mathrm{g}$.

Hydrolysis is acid catalyzed ${ }^{5}$ and at low [A] obeys the equations ${ }^{6}$

$$
\begin{align*}
& (d[\mathrm{~A}] / d t)=k\left[\mathrm{E}_{0}\right]\left[\mathrm{W}_{0}\right][\mathrm{A}]  \tag{2}\\
& \ln \left([\mathrm{A}] /\left[\mathrm{A}_{0}\right]\right)=k\left[\mathrm{E}_{0}\right]\left[\mathrm{W}_{0}\right] t \tag{3}
\end{align*}
$$

where $t$ is time and the 0 subscripts indicate use of initial concentrations. Plots of $\ln \left([A] /\left[\mathrm{A}_{0}\right]\right)$ vs. $t$ are linear between $\left[\mathrm{A}_{0}\right]$, about $10^{-5} \mathrm{~mol} / \mathrm{g}$, and $[\mathrm{A}]$, equal to about $5 \times 10^{-4} \mathrm{~mol} / \mathrm{g}$. Such linearity appears reasonable over this range of [A], because [E] decreases only about $5 \%$ from its initial value of about $10^{-2} \mathrm{~mol} / \mathrm{g}$, and [W], which was measured in polyester polyurethanes, does not change significantly with hydrolysis. ${ }^{6}$ Slopes of the $\ln \left([\mathrm{A}] /\left[\mathrm{A}_{0}\right]\right)$ vs $t$ plots are equal to $k\left[\mathrm{E}_{0}\right]\left[\mathrm{W}_{0}\right]$. The values of this product are approximately proportional to RH. ${ }^{6}$ $\left[\mathrm{E}_{0}\right]$ ( $\mathrm{mol} / \mathrm{g}$ ) is just the reciprocal of the molecular weight of the repeating ester unit multiplied by the weight fraction of ester. Hence $k\left[\mathrm{~W}_{0}\right]$ can be calculated.

Eventually the back reaction becomes important. Then one expects that

$$
\begin{equation*}
(d[\mathrm{~A}] / d t)=k[\mathrm{~W}][\mathrm{E}][\mathrm{A}]-k^{\prime}[\mathrm{A}]^{2}[\mathrm{H}] \tag{4}
\end{equation*}
$$

Here the term for the rate of the back reaction includes an $[A]^{2}$, because the acid catalyst for the forward reaction ought to catalyze the back reaction also. The 1:1 stoichiometry in reaction (1) indicates that $[\mathrm{E}]=\left[\mathrm{E}_{0}\right]-[\mathrm{A}]$ and $[\mathrm{H}]=\left[\mathrm{H}_{0}\right]$ $+[A]$. The polyesters used were initially diols so that $\left[\mathrm{H}_{0}\right]$ for each is twice the reciprocal of its number average molecular weight $M_{n}$.

If $k[\mathrm{~W}]=k\left[\mathrm{~W}_{0}\right]$ throughout, the only unknown is $k^{\prime}$. This can be calculated from the equilibrium acid concentration $\left[\mathrm{A}_{e}\right]$ since then $(d[\mathrm{~A}] / d t)=0$ and

$$
\begin{equation*}
k^{\prime}=\frac{k\left[\mathrm{~W}_{0}\right]\left(\left[\mathrm{E}_{0}\right]-\left[\mathrm{A}_{e}\right]\right)}{\left[\mathrm{A}_{e}\right]\left(\left[\mathrm{H}_{0}\right]+\left[\mathrm{A}_{e}\right]\right)} \tag{5}
\end{equation*}
$$

If similar substitutions are made for $[\mathrm{E}],[\mathrm{H}]$, and [W] in eq. (4) and the variables are separated, one obtains

$$
\begin{equation*}
d t=\frac{d[\mathrm{~A}]}{[\mathrm{A}]\left(-k^{\prime}[\mathrm{A}]^{2}-\left(k\left[\mathrm{~W}_{0}\right]+k^{\prime}\left[\mathrm{H}_{0}\right]\right)[\mathrm{A}]+k\left[\mathrm{~W}_{0}\right]\left[\mathrm{E}_{0}\right]\right)} \tag{6}
\end{equation*}
$$

This has the reduced form $(d x / x X)$, where $x=[\mathrm{A}]$ and $X=a x^{2}+b x+c$. Here $a=-k^{\prime}, b=-\left(k\left[\mathrm{~W}_{0}\right]+k^{\prime}\left[\mathrm{H}_{0}\right]\right)$, and $c=k\left[\mathrm{~W}_{0}\right]\left[\mathrm{E}_{0}\right]$. After integration

$$
\begin{equation*}
t=\left[\frac{1}{2 c} \ln \left|\frac{x^{2}}{X}\right|-\frac{b}{2 a c(p-q)} \ln \left|\frac{x-p}{x-q}\right|\right]_{x_{0}}^{x} \tag{7}
\end{equation*}
$$

Here, $p$ and $q$ are roots of $X=0$.
Thus, from measurements of $\left[\mathrm{A}_{e}\right]$ and the initial rate of acid formation, one can calculate times required to reach definite acid concentrations. It will be seen that calculated and observed times usually agree well at $95 \%, 50 \%$, and $25 \% \mathrm{RH}$ in polyesters but that $k^{\prime}$ increases markedly as RH decreases.

It was found that [W] did depend on [A] in polyesters, particularly at high RH. Some of the apparent variation in $k^{\prime}$ was eliminated by treating [W] as a variable. Modifications to the equations that accomplish this are developed later.
$[\mathrm{H}]$ in polyester polyurethanes is different from values in polyester diols because the isocyanate used to form the urethane does so by reacting with alcohol groups. We calculated [ $\mathrm{H}_{0}$ ] as follows. $M_{n}$ of the polyurethanes before any aging was about $20,000,{ }^{6}$ giving about $10^{-4} \mathrm{~mol}$ chain ends $/ \mathrm{g}$. The initial acid content was about $10^{-5} \mathrm{~mol} / \mathrm{g}$. Therefore, initially $10 \%$ of the chain ends were acid groups and so presumably the rest were alcohol groups. Consequently, $\left[\mathrm{H}_{0}\right]$ for polyester polyurethanes will be taken as equal to $10^{-4} \mathrm{~mol} / \mathrm{g}$. Equations (6) and (7) should describe the equilibration of these polymers except at $0 \% \mathrm{RH}$. Under that condition there is no forward reaction and the applicable equation is

$$
\begin{equation*}
-(d[\mathrm{~A}] / d t)=k^{\prime}[\mathrm{A}]^{2}\left(\left[H_{0}\right]+[\mathrm{A}]\right) \tag{8}
\end{equation*}
$$

After integration

$$
\begin{equation*}
t=\frac{1}{k^{\prime}}\left[\frac{1}{\left[\mathrm{H}_{0}\right][\mathrm{A}]}-\frac{1}{\left[\mathrm{H}_{0}\right]^{2}} \ln \left(\frac{\left[\mathrm{H}_{0}\right]+[\mathrm{A}]}{[\mathrm{A}]}\right)\right]_{\left[\mathrm{A}_{0}\right]}^{[\mathrm{A}]} \tag{9}
\end{equation*}
$$

## EXPERIMENTAL

The materials studied and their $\left[\mathrm{E}_{0}\right],\left[\mathrm{H}_{0}\right]$, and $\left[\mathrm{A}_{0}\right]$ are listed in Table I. Additional data on these materials have been published. ${ }^{6,7} 63 \mathrm{PBA}$ is designated 63 PTMA in Ref. 6. No catalyst was used in preparing PBA 930. A tin ester catalyst was used to make PBA 1090. The high [ $\mathrm{A}_{0}$ ] of 71 PCL and 63 PBA were brought about by preaging at high RH.

The polyester diols were aged in vessels sketched in Figure 1. These were of glass, except for a Teflon-glass valve at the top. About 10 g of polyester diol were placed in the inner dish. Preliminary experiments had shown that $2-10 \mathrm{~g}$ in the inner dish gave the same rates of acid formation, indicating that reaction was

TABLE I
Materials Studied and Their Initial Properties

|  |  | Concentration (mol/g) |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Material | Designation | $10^{3}\left[\mathrm{E}_{0}\right]$ | $10^{3}\left[\mathrm{H}_{0}\right]$ | $10^{5}\left[\mathrm{~A}_{0}\right]$ |
| Polycaprolactone diol, $M_{n}=2000$ | PCL 2000 | 8.77 | 1.00 | 0.74 |
| Polyurethane with 71 wt $\%$ PCL | 71 PCL | 6.2 | 0.1 | 25.5 |
| Poly(butylene adipate) diol, $M_{n}=930$ | PBA 930 | 10 | 2.15 | 8.4 |
| Poly(butylene adipate) diol, $M_{n}=1090$ | PBA 1090 | 10 | 1.83 | 0.42 |
| Polyurethane with 63 wt $\%$ PBA | 63 PBA | 6.3 | 0.1 | 35.4 |



Fig. 1. Equilibration vessel.
not diffusion-controlled. Beneath the inner dish were $50 \mathrm{~cm}^{3}$ of an aqueous solution of LiCl . Concentrations were used that gave about $95 \%, 50 \%, 25 \% \mathrm{RH}$. Published data were used to calculate RH. ${ }^{8}$ After a brief evacuation to eliminate oxygen, tubes were filled with argon and immersed in an oil bath at $85^{\circ} \mathrm{C}$ to a level above the bottom of the valve stem. The stem was removed at intervals, and samples of polyester were withdrawn using a preheated pipette.

Acid contents of these samples were measured by titration, as described earlier. ${ }^{6}$ Water contents [W] were measured by diluting samples 10 -fold with dry tetrahydrofuran and injecting $10-\mathrm{mL}$ portions into a gas chromatograph having a Poropak column kept at $140^{\circ} \mathrm{C}$. The height of the water peak was used to determine the water content, by comparison with a calibration which was constructed by injecting water-tetrahydrofuran mixtures of known concentration. This method is not very precise, since deviations between replicate injections averaged about $15 \%$. The dependence of [W] on [A] can be strong so the data were adequate for our purpose.

Equilibration was assumed to have occurred when the acid content became nearly constant. Then the vessel usually was transferred to a $55^{\circ} \mathrm{C}$ oil bath and additional samples were taken. When equilibration had occurred, the solution of LiCl was removed. Its density was measured to check the concentration, which was changed slightly from the initial value because water is sorbed by the ester and consumed by hydrolysis. Back reactions were studied at $85^{\circ} \mathrm{C}$ by adding more concentrated LiCl solutions to tubes that had been equilibrated at $95 \%$ or $50 \% \mathrm{RH}$. Thus equilibrium was approached from both sides at $50 \%$ and $25 \%$ RH.

Erratic values of $\left[\mathrm{A}_{e}\right]$ were obtained at $100 \% \mathrm{RH}$. Under this condition an
aqueous phase eventually collected in the inner dish, probably because of thermal gradients in the bath. To explore this effect, diols and water in equal weights were stirred together at $60^{\circ} \mathrm{C}$ and then separated. Evaporation of the water from the aqueous layer left a residue of low molecular weight polyester. About 5\% and $1 \%$ water soluble materials were found in polycaprolactone diols of $M_{n} 540$ and 2000 , respectively. The soluble material from the former polyester was redissolved in water and aged at $85^{\circ} \mathrm{C}$. It formed acid with a rate constant about three times that found initially for the bulk diol exposed to water vapor only. Therefore, we suspect that if an aqueous liquid phase contacts these polyesters, low molecular weight organic components dissolve in the water, and form acid rapidly. This diffuses into the organic phase, augmenting the acid content there. Gel permeation chromatograms confirmed that it is the low molecular weight components from the polyester diols that are water-soluble. Only a single phase in the inner dish was observed at $95 \%, 50 \%$, and $25 \% \mathrm{RH}$, and so these are the RH for which we report equilibrium data.

Polyester-polyurethanes were aged in sealed glass tubes containing aqueous LiCl solutions giving about $25 \%$ and $10 \% \mathrm{RH}$. Our interest in the polymers was primarily in their behavior at low RH so equilibration was not attempted at high RH. Reaction at zero RH was studied by pumping on thermostatted samples. Volatiles evolved for several days, as evidence by condensation outside the baths. [ $\mathrm{A}_{0}$ ] was taken as the value after volatilization appeared to have ended. Molecular weights of some polyester-polyurethanes were determined by gel chromatography, as described before. ${ }^{6}$

## RESULTS

## Polyesters

The equilibrium acid contents of the polyesters at $85^{\circ} \mathrm{C}$ are listed in Table II along with the final RH and an indication of whether the results came from a

TABLE II
Equilibration of Polyesters at $85^{\circ} \mathrm{C}$

| Polyester <br> (Fig.) | RH <br> $(\%)$ | $\mathrm{F} / \mathrm{B}$ | $10^{3}\left[\mathrm{~A}_{e}\right]$ <br> $(\mathrm{mol} / \mathrm{g})$ | Line | $k\left[\mathrm{~W}_{0}\right]$ <br> $(\mathrm{g} / \mathrm{mol} \cdot$ day $)$ | $10^{-4} k^{\prime}$ <br> $\left(\mathrm{g}^{2} / \mathrm{mol}^{2} \cdot\right.$ day $)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PCL 2000 (2) | 93 | F | 3.16 | 1 | 23.7 | 1.01 |
|  | 53 | B | 1.33 | 2 | 13.3 | 3.19 |
|  | 50 | F | 1.25 | 3 | 13.0 | 3.50 |
|  | 25 | B | 0.62 | 4 | 6.8 | 5.52 |
|  | 25 | F | 0.57 | 5 | 6.8 | 6.23 |
| PBA 930 (3) | 93 | F | 3.95 | 1 | 21.8 | 0.55 |
|  | 53 | B | 1.52 | 2 | 13.2 | 2.01 |
|  | 50 | F | 1.24 | 3 | 12.3 | 2.57 |
|  | 25 | B | 0.62 | 4 | 6.0 | 3.29 |
|  | 25 | F | 0.76 | 5 | 6.0 | 2.56 |
|  | 93 | F | 3.98 | 1 | 27.0 | 0.70 |
|  | 53 | B | 1.54 | 2 | 15.2 | 2.48 |
|  | 50 | F | 1.38 | 3 | 14.5 | 2.82 |
|  | 25 | B | 0.67 | 4 | 10.5 | 5.85 |
|  | 25 | F | 0.75 | 5 | 10.5 | 5.02 |



Fig. 2. Equilibration of PCL 2000 at $85^{\circ} \mathrm{C}(+, 1), 93 \% \mathrm{RH} ;(\Delta, 3 ; \Delta, 2) \mathrm{F}$ and B reactions, $50 \%$ and $53 \% \mathrm{RH}$; $(\bullet, 5 ; \diamond, 4) \mathrm{F}$ and B reactions, $25 \% \mathrm{RH}$. Lines by eq. (7) with the $k\left[\mathrm{~W}_{0}\right]$ and $k^{\prime}$ in Table II.
forward (F) or back (B) reaction. Kinetic data are shown in figures whose numbers appear under the listed polyesters. Also listed are line numbers and values of $k\left[\mathrm{~W}_{0}\right]$ and $k^{\prime}$ used in calculating each line in these figures by eq. (7). $\left[\mathrm{E}_{0}\right],\left[\mathrm{H}_{0}\right]$, and $\left[\mathrm{A}_{0}\right]$ for forward reactions, are the values in Table I. [ $\left.\mathrm{A}_{0}\right]$ for each back reaction is $\left[\mathrm{A}_{e}\right]$ for the previous forward reaction. $k\left[\mathrm{~W}_{0}\right]$ in back reactions are those in corresponding forward reactions, scaled proportional to RH if RH was slightly different.

The largest $\left[\mathrm{A}_{e}\right]$, about $4 \times 10^{-3} \mathrm{~mol} / \mathrm{g}$ is about $40 \%$ of $\left[\mathrm{E}_{0}\right]$, so it is worthwhile to use eq. (7), which allows for the change in [E]. Each ester group hydrolyzed also represents a chain scission, so that $M_{n}$ of the polyester can be as low as 250 at equilibrium. [ $\mathrm{A}_{e}$ ] changes about $4 \times 10^{-5} \mathrm{~mol} / \mathrm{g}$ for each percent change in RH . $\left[\mathrm{A}_{e}\right]$ at $55^{\circ} \mathrm{C}^{1}$ are not very different from those at $85^{\circ} \mathrm{C}$, and so the former are not shown here.


Fig. 3. Equilibration of PBA 930 at $85^{\circ} \mathrm{C}$. Symbols and lines as in Figure 2.


Fig. 4. Equilibration of PBA 1090 at $85^{\circ} \mathrm{C}$. Symbols and lines as in Figure 2.

The titration is such that $\left[\mathrm{A}_{e}\right]$ ought to be good to $\sim 1-2 \%$ of itself. RH at equilibrium should be known to about $1 \%$. Differences between $\left[\mathrm{A}_{e}\right]$ in forward and back reactions, after correction for the small differences in RH , exceed these uncertainties in PBA 930 and PBA 1090 at $50 \% \mathrm{RH}$, and in all three polyesters at $25 \% \mathrm{RH}$. Causes of these unexpectedly large discrepancies are not known.

Each line in Figures 2-4 is a computer-driven plot of $t$, [A] that was obtained by using eq. (7) to calculate $t$ at 50 values of $[\mathrm{A}]$ between $\left[\mathrm{A}_{0}\right]$ and $\left[\mathrm{A}_{e}\right]$. The lines are composed of straight segments joining the $t,[\mathrm{~A}]$. This segmental construction accounts for occasional abrupt changes in slope.

Values of $k\left[\mathrm{~W}_{0}\right]$ are larger in PBA 1090 than in PBA 930, particularly at $25 \%$ RH. The esterification catalyst in PBA 1090 may account for the differences. Possible such catalysts are themselves subject to hydrolysis, which would complicate the situation.


Fig. 5. Water content of polyesters at $55^{\circ} \mathrm{C}$ and $85^{\circ} \mathrm{C}$, as a function of acid content: ( $\boldsymbol{\square}, 55^{\circ} \mathrm{C}$; $(\Delta, \square) 85^{\circ} \mathrm{C} ;(\boldsymbol{\Delta}, \Delta) \mathrm{PCL} 2000 ;(\square, \square)$ PBA 930 and PBA 1090; (1) $[\mathrm{W}]=614[\mathrm{~A}]^{2}+0.0029$; (2) $[\mathrm{W}]$ $=2.09[\mathrm{~A}]+0.0029 ;(3)[\mathrm{W}]=555[\mathrm{~A}]^{2}+0.002 ;(4)[\mathrm{W}]=1.61[\mathrm{~A}]+0.002$.


Fig. 6. Equilibration of PBA 930 at $85^{\circ} \mathrm{C}, 93 \% \mathrm{RH}:(+)$ experimental data; (-) eq. (7), $[\mathrm{W}]=$ $\left[\mathrm{W}_{0}\right] ;(\cdots \cdot)$ eq. $\left(7^{\prime}\right),[\mathrm{W}]=2.09[\mathrm{~A}]+0.0029 ;(---)$ integral of eq. $\left(6^{\prime \prime}\right),[\mathrm{W}]=614[\mathrm{~A}]^{2}+0.0029$.
$k^{\prime}$ increases by factors of $6-8$ for these polyesters as RH decreases from $93 \%$ to $25 \%$. This was surprising since [W] is not an explicit factor in the back reaction. Consequently, [W] was determined at various [A] because the observed variation in $k^{\prime}$ might be caused by changes in $k[\mathrm{~W}]$ with [A]. Figure 5 is a plot of [W] vs. [A] for results at the $85^{\circ} \mathrm{C}, 93 \% \mathrm{RH}$ and $55^{\circ} \mathrm{C}, 89 \% \mathrm{RH}$. There is about a fourfold increase in water content between $\left[\mathrm{A}_{0}\right]$ and $\left[\mathrm{A}_{e}\right]$, so that $k[\mathrm{~W}]$ presumably increases by the same factor. It is convenient to represent the solubility by analytical functions in order to modify eqs. (4)-(7). Quadratic and linear variations are shown in Figure 5, one of each for PCL 2000 and one each for the PBA polyesters.

The water content is almost $20 \mathrm{wt} \%$ when $[\mathrm{A}]=4 \times 10^{-3} \mathrm{~mol} / \mathrm{g}$. Therefore, in addition to substituting functions for [W] in eq. (4), one ought to allow for the dilution of ester and hydroxyl by water. This can be done by multiplying $\mathrm{E}_{0}$ and $\mathrm{H}_{0}$ by (1-18[W]) because $18[\mathrm{~W}]$ represents the weight of water in each gram of polyester and dissolved water. [A] need not be multiplied by this factor because any dissolved water was included in the weight of samples used in the titrations by which [A] was determined.

Let constants in the equations relating [W] and [A] be $k^{\prime \prime}$ and [ $\mathrm{W}_{0}$ ], i.e.: linear variation, $[\mathrm{W}]=k^{\prime \prime}[\mathrm{A}]+\left[\mathrm{W}_{0}\right]$; quadratic variation, $[\mathrm{W}]=\mathrm{k}^{\prime \prime}[\mathrm{A}]^{2}+\left[\mathrm{W}_{0}\right]$. If linear variation is correct,

$$
k^{\prime}=\frac{k\left(k^{\prime \prime}\left[\mathrm{A}_{e}\right]+\left[W_{0}\right]\right)\left(\left[E_{0}\right]-18\left[\mathrm{E}_{0}\right] k^{\prime \prime}\left[\mathrm{A}_{e}\right]-18\left[\mathrm{E}_{0}\right]\left[\mathrm{W}_{0}\right]-\left[\mathrm{A}_{e}\right]\right)}{\left[A_{e}\right]\left(\left[\mathrm{H}_{0}\right]-18\left[\mathrm{H}_{0}\right] k^{\prime \prime}\left[\mathrm{A}_{e}\right]-18\left[\mathrm{H}_{0}\right]\left[\mathrm{W}_{0}\right]+\left[\mathrm{A}_{e}\right]\right)}
$$

Equations for $d t,\left(6^{\prime}\right)$, and $t,\left(7^{\prime}\right)$, will not be written out because they have the same reduced forms as eqs. (6) and (7), respectively, but with

$$
\begin{gathered}
a=-18\left[\mathrm{E}_{0}\right] k\left(k^{\prime \prime}\right)^{2}-k k^{\prime \prime}-k^{\prime}+18 k^{\prime} k^{\prime \prime}\left[\mathrm{H}_{0}\right] \\
b=k k^{\prime \prime}\left[\mathrm{E}_{0}\right]-36 k k^{\prime \prime}\left[\mathrm{E}_{0}\right]\left[\mathrm{W}_{0}\right]-k\left[\mathrm{~W}_{0}\right]-k^{\prime}\left[\mathrm{H}_{0}\right]+18 k^{\prime}\left[\mathrm{H}_{0}\right]\left[\mathrm{W}_{0}\right] \\
c=k\left[\mathrm{E}_{0}\right]\left(\left[\mathrm{W}_{0}\right]-18\left[\mathrm{~W}_{0}\right]^{2}\right)
\end{gathered}
$$



Fig. 7. Equilibration of PBA 930 at $85^{\circ} \mathrm{C}, 53 \%$ and $50 \% \mathrm{RH}$ : (■, व) experimental data for F and $B$ reactions; (—) eq. (7), $[\mathrm{W}]=\left[\mathrm{W}_{0}\right] ;(\cdots .$.$) eq. (7), [\mathrm{W}]=0.55[\mathrm{~A}]+6.4 \times 10^{-4}$.

If quadratic variation is correct,

$$
\begin{gather*}
k^{\prime}=\frac{k\left(k^{\prime \prime}\left[\mathrm{A}_{e}\right]^{2}+\left[\mathrm{W}_{0}\right]\right)\left(\left[\mathrm{E}_{0}\right]-18\left[\mathrm{E}_{0}\right] k^{\prime \prime}\left[\mathrm{A}_{e}\right]^{2}-18\left[\mathrm{E}_{0}\right]\left[\mathrm{W}_{0}\right]-\left[\mathrm{A}_{e}\right]\right)}{\left(\left[\mathrm{H}_{0}\right]-18\left[\mathrm{H}_{0}\right] k^{\prime \prime}\left[\mathrm{A}_{e}\right]^{2}-18\left[\mathrm{H}_{0}\right]\left[\mathrm{W}_{0}\right]+\left[\mathrm{A}_{e}\right]\right)\left[\mathrm{A}_{e}\right]} \\
d t=\frac{d[\mathrm{~A}]}{Q_{5}[\mathrm{~A}]^{5}+Q_{4}[\mathrm{~A}]^{4}+Q_{3}[\mathrm{~A}]^{3}+\mathrm{Q}_{2}[\mathrm{~A}]^{2}+\mathrm{Q}_{1}[\mathrm{~A}]}
\end{gather*}
$$

where

$$
\begin{aligned}
& Q_{5}=-18 k\left(k^{\prime \prime}\right)^{2}\left[\mathrm{E}_{0}\right] \\
& Q_{4}=18 k^{\prime} k^{\prime \prime}\left[\mathrm{H}_{0}\right]-k k^{\prime \prime} \\
& Q_{3}=k k^{\prime \prime}\left[\mathrm{E}_{0}\right]-36 k k^{\prime \prime}\left[\mathrm{E}_{0}\right]\left[\mathrm{W}_{0}\right]-k^{\prime} \\
& Q_{2}=18 k^{\prime}\left[\mathrm{H}_{0}\right]\left[\mathrm{W}_{0}\right]-k\left[\mathrm{~W}_{0}\right]-k^{\prime}\left[\mathrm{H}_{0}\right] \\
& Q_{1}=k\left[\mathrm{E}_{0}\right]\left[\mathrm{W}_{0}\right]-18 k\left[\mathrm{E}_{0}\right]\left[\mathrm{W}_{0}\right]^{2}
\end{aligned}
$$

No attempt was made to integrate eq. (6") to give an expression in closed form. Instead, a prepackaged trapezoidal integration program ${ }^{9}$ was used to calculate $t$ at 50 values of $[\mathrm{A}]$ between $\left[\mathrm{A}_{0}\right]$ and $\left[\mathrm{A}_{e}\right]$.

Figure 6 compares the three calculation methods with one another and with the experimental data for PBA 930 at $93 \% \mathrm{RH}$. Linear variation of [W] with [A] gives a more rapid rise of [A] with $t$ than observed. There is about as good agreement between the experimental data and the line with [ W ] independent of [A] as there is with the line for quadratic variation of [W] with [A].

Water solubilities were also determined at $50 \%$ and $25 \%$ RH. Quadratic relationships between [W] and [A] did not give as good a fit to kinetics as linear relationships. ${ }^{1}$ Therefore, we use only the latter here. Data for the three polyesters at $50 \%$ and $25 \% \mathrm{RH}$ were represented by the equations $[\mathrm{W}]=0.55[\mathrm{~A}]$ $+6.4 \times 10^{-4}$ and $[\mathrm{W}]=0.17[\mathrm{~A}]+2.5 \times 10^{-4}$, respectively. Figures 7 and 8 compares the experimental [A], $t$ data for PBA 930 at $50 \%$ and $25 \% \mathrm{RH}$ with values calculated by eqs. (7) and ( $7^{\prime}$ ). There is very little difference between results calculated by these equations.


Fig. 8. Equilibration of PBA 930 at $85^{\circ} \mathrm{C}, 25 \% \mathrm{RH}$ : ( $\square$, ロ) experimental data for F and B reactions; $(-)$ eq. $(7),[W]=\left[W_{0}\right] ;(\cdots \cdot)$ eq. $\left(7^{\prime}\right),[W]=0.17[A]+2.5 \times 10^{-4}$.

Similar calculations were made for PBA 1090 and PCL 2000. Results are qualitatively similar to those in Figures 6-8.

Table III contains the $k$ and $k^{\prime}$ that were used in these calculations. Both $k$ and $k^{\prime}$ increase by factor of $3-4$ as RH decreases. Thus allowance for the dependence of [W] on [A] reduces but does not eliminate changes in $k^{\prime}$ with RH

TABLE III

| $k$ and $k^{\prime}$ with [W] Dependent on [A] ${ }^{\text {a }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Polyester | RH |  | $10^{-4} k$ | $10^{-4} k^{\prime}$ |
|  | (\%) | F/B | $\left(\mathrm{g}^{2} / \mathrm{mol}^{2} \cdot \mathrm{day}\right)$ |  |
| PCL 2000 | 93 | F | $1.19{ }^{\text {b }}$ | $3.11^{\text {b }}$ |
|  | 93 | F | 1.19 | 2.96 |
|  | 53 | B | 2.08 | 6.72 |
|  | 50 | F | 2.03 | 7.08 |
|  | 25 | B | 2.72 | 7.82 |
|  | 25 | F | 2.72 | 8.62 |
| PBA 930 | 93 | F | $0.75{ }^{\text {c }}$ | $1.61{ }^{\text {c }}$ |
|  | 93 | F | 0.75 | 1.61 |
|  | 53 | B | 2.06 | 4.55 |
|  | 50 | F | 1.92 | 5.22 |
|  | 25 | B | 2.40 | 4.65 |
|  | 25 | F | 2.40 | 4.65 |
| PBA 1090 | 93 | F | $0.93{ }^{\text {c }}$ | $2.05{ }^{\text {c }}$ |
|  | 93 | F | 0.93 | 1.93 |
|  | 53 | B | 2.38 | 5.67 |
|  | 50 | F | 2.27 | 6.09 |
|  | 25 | B | 4.20 | 8.49 |
|  | 25 | F | 4.20 | 7.56 |

[^1]

Fig. 9. Acid content vs. time in 63 PBA and 71 PCL. (1-4) by eq. (9); (5-7) by eq. (7). ( $\uparrow, \mathbf{4}$, ■, $\diamond, \Delta, \square,+$ ) experimental data. $(\bullet, \Delta, \square,+) 0 \% \mathrm{RH}:(\diamond) 71 \mathrm{PCL}, 85^{\circ} \mathrm{C} ;(\Delta) 63 \mathrm{PBA}, 85^{\circ} \mathrm{C}$; ( $\boldsymbol{\square}$ ) $63 \mathrm{PBA}, 55^{\circ} \mathrm{C} ;(+) 63 \mathrm{PBA}, 30^{\circ} \mathrm{C}$. $(\diamond, \Delta, \square) 25 \% \mathrm{RH}$ : ( $\left.\diamond\right) 71 \mathrm{PCL}, 85^{\circ} \mathrm{C}$; ( $\Delta$ ) $63 \mathrm{PBA}, 85^{\circ} \mathrm{C}$; ( $\square$ ) 63 PBA, $55^{\circ} \mathrm{C}$.
and indicates that $k$ depends on RH. Of course, it must really be the change in [W] with RH that affects $k$ and $k^{\prime}$. Later we speculate on how [W] can affect $k$ and $k^{\prime}$. Since [W] also depends on [A], $k$ and $k^{\prime}$ presumably change during reaction, contrary, to all of our equations. Thus the generally good agreement between our kinetic approaches and the experimental data is rather surprising, especially since $k$ is calculated from data at low $[\mathrm{A}]$ and $k^{\prime}$ is calculated from data at high [A].

## Polyester-Polyurethanes

Figure 9 shows data obtained with polyester-polyurethanes. Most of the experiments involved 63 PBA because PBA elastomers are used in making tape binder. Lines $1-4$ are plots of eq. (9) $(0 \% \mathrm{RH})$ using $k^{\prime}$ calculated from the [ $\mathrm{A}_{0}$ ] found when the rate of weight loss had become negligible and the [A] at the longest time at each temperature. Lines 5-7 are plots of eq. (7) for results obtained at about $25 \% \mathrm{RH} . k\left[\mathrm{~W}_{0}\right]$ was calculated by eq. (3) from $\left[\mathrm{A}_{0}\right]$ and $t,[\mathrm{~A}]$ at the first analysis. Lines 5,6 , and 7 lie below these first points because the back reaction is appreciable. Values of the rate constants and $\left[\mathrm{A}_{\mathrm{e}}\right]$ are listed in Table IV.

These $\left[\mathrm{A}_{e}\right]$ are really subjective estimates since [A] never became constant. A value of $8 \times 10^{-4} \mathrm{~mol} / \mathrm{g}$ for 63 PBA at $25 \% \mathrm{RH}$ and $85^{\circ} \mathrm{C}$ seemed reasonable since the rate of increase in $[\mathrm{A}]$ had become much less than it was initially. [ $\left.\mathrm{A}_{e}\right]$ $=8 \times 10^{-4} \mathrm{~mol} / \mathrm{g}$ was also used for 63 PBA at $25 \% \mathrm{RH}$ and $55^{\circ} \mathrm{C}$ because values of $\left[\mathrm{A}_{e}\right]$ of the polyesters were independent of temperature. Successive values of [A] in 71 PCL were nearly equal to $6 \times 10^{-4} \mathrm{~mol} / \mathrm{g}$, and so we chose this value as $\left[\mathrm{A}_{e}\right]$. The values of $k\left[\mathrm{~W}_{0}\right]$ and $k^{\prime}$ for 63 PBA at $25 \% \mathrm{RH}, 85^{\circ} \mathrm{C}$ are less than found for PBA 930 and PBA 1090 but [ $\mathrm{A}_{e}$ ] for the latter are kept lower by the larger value of $\left[\mathrm{H}_{0}\right]$. Values of $k^{\prime}$ at $0 \% \mathrm{RH}, 85^{\circ} \mathrm{C}$ much exceed the values at $25 \%$

TABLE IV
63 PBA and 71 PCL, Constants Used in Calculations

|  | Polymer | Temp <br> $\left({ }^{\circ} \mathrm{C}\right)$ | RH <br> $(\%)$ | $10^{4}\left[\mathrm{~A}_{e}\right]$ <br> $(\mathrm{mol} / \mathrm{g})$ | $k\left[\mathrm{~W}_{0}\right]$ <br> $(\mathrm{g} / \mathrm{mol} \cdot$ day $)$ | $10^{-5} k^{\prime}$ <br> $\left(\mathrm{g}^{2} / \mathrm{mol}^{2} \cdot\right.$ day $)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 71 PCL | 85 | 0 |  |  | 2.71 |
| 2 | 63 PBA | 85 | 0 |  |  | 1.93 |
| 3 | 63 PBA | 55 | 0 |  |  | 0.185 |
| 4 | 63 PBA | 30 | 0 |  |  | 0.030 |
| 5 | 63 PBA | 55 | 25 | $\sim 8$ | 0.438 | 0.033 |
| 6 | 71 PCL | 85 | 25 | $\sim 6$ | 5.33 | 0.688 |
| 7 | 63 PBA | 85 | 25 | $\sim 8$ | 3.52 | 0.269 |

RH. Thus, $k^{\prime}$ appears to depend on humidity in polyurethanes as well as in polyesters.
$M_{n}$ was determined for preaged 63 PBA and for samples aged at $0 \% \mathrm{RH}$ at $85^{\circ} \mathrm{C}$ for 70 days and at $55^{\circ} \mathrm{C}$ for 164 days. Respective $M_{n}$ are 2650,5350 , and 3750. Thus $M_{n}$ is indeed increasing during aging at $0 \% \mathrm{RH}$, and the changes are larger the greater the change in [A]. None of the aged samples could be considered elastomeric since they broke on small extensions. The rate of change in [A] presumably decreases with [A] according to eq. (9) so that it does not seem practical to regenerate elastomers by reversing hydrolysis.

Additional data obtained with 63 PBA and 71 PCL are in Table V. Most of these data are from experiments at about $10 \% \mathrm{RH}$. The acid content of 63 PBA increases at $\sim 10 \%, \mathrm{RH}, 85^{\circ} \mathrm{C}$ but appears to decrease very slightly at $55^{\circ} \mathrm{C}$ and $35^{\circ} \mathrm{C}$. [A] increases slightly at $23 \% \mathrm{RH}, 35^{\circ} \mathrm{C}$. Therefore, $\left[\mathrm{A}_{e}\right]$ is $\sim 3.5 \times 10^{-4}$ $\mathrm{mol} / \mathrm{g}$ at $\sim 18 \% \mathrm{RH}, 35^{\circ} \mathrm{C}$. The rates of change in [A] are so slow that very precisely determined $[\mathrm{A}]$ would be required to locate true equilibrium.

Results listed for 71 PC at $\sim 10 \% \mathrm{RH}$, are more erratic but show qualitatively the same behavior as those for 63 PBA , an increase in [A] at $85^{\circ} \mathrm{C}$ and little change at $55^{\circ} \mathrm{C}$ and $35^{\circ} \mathrm{C}$.

## DISCUSSION

To us it appears unlikely that equilibrium limits hydrolysis in a practical sense in thermoplastic elastomers. Values of $\left[\mathrm{A}_{e}\right]$ must approach $5 \times 10^{-4} \mathrm{~mol} / \mathrm{g}$ even at $25 \% \mathrm{RH}$ and probably more than $10^{-3} \mathrm{~mol} / \mathrm{g}$ at $50 \% \mathrm{RH}$. This is much more acid than present normally, indicating that enough scissions should have occurred by reaction 1 so that $M_{n}\left[\mathrm{~A}_{e}\right] \sim 1 \mathrm{~mol}$ acid/polymer molecule. Thus $M_{n} \sim\left[\mathrm{~A}_{e}\right]^{-1}$ or $\sim 2000$ at $25 \%$ RH and less at higher RH. Materials of such molecular weights are not elastomeric.

The presence of crosslinks could change this conclusion. Random degradation generates a most probable distribution. Therefore, an $M_{n}$ of $1000-2000$ means that the weight average molecular weight $M_{w}$ is $2000-4000$. One crosslinked unit per weight average molecule represents the gel point so that a polymer with an initial crosslink density of $2.5-5 \times 10^{-4} \mathrm{~mol} / \mathrm{g}$ presumably would just have gel at $\left[\mathrm{A}_{e}\right]=5 \times 10^{-4}$ to $10^{-3} \mathrm{~mol} / \mathrm{g}$. Somewhat more crosslinks might be desirable, presumably, but a practical upper limit would be to crosslink all the urethane groups, giving about one crosslinked unit in 10 ester groups.

TABLE V
Aging of Preaged 63 PBA and 71 PCL at Low RH

| days | $10^{4}[\mathrm{~A}](\mathrm{mol} / \mathrm{g})$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $12 \% \mathrm{RH}, 85^{\circ} \mathrm{C}$ | 10\% RH, $55^{\circ} \mathrm{C}$ | $12 \% \mathrm{RH}, 35^{\circ} \mathrm{C}$ | $23 \% \mathrm{RH}, 35^{\circ} \mathrm{C}$ |
| 63 PBA |  |  |  |  |
| 0 | 3.54 | 3.54 | 3.54 | 3.54 |
| 14 | 3.84 |  |  |  |
| 36 | 3.93 | 3.41 |  |  |
| 76 | 4.40 | 3.43 |  | 3.57 |
| 120 | 4.86 |  |  |  |
| 164 | 5.33 | 3.39 | 3.38 | 3.68 |
| 298 |  |  | 3.35 | 3.84 |
| 71 PCL |  |  |  |  |
| 0 | 2.55 | 2.55 | 2.55 |  |
| 14 | 3.21 |  |  |  |
| 38 | 4.37 | 3.20 | 2.60 |  |
| 58 | 3.51 |  |  |  |
| 105 |  | 3.33 |  |  |
| 123 |  |  | 2.81 |  |
| 158 |  | 2.89 |  |  |
| 176 |  |  | 2.67 |  |
| 252 |  | 2.96 | 2.56 |  |

There might also be a kinetic factor giving a lower $\left[\mathrm{A}_{e}\right]$ in a gel than in an uncrosslinked polymer. Acid and hydroxyl groups must separate by diffusion for a hydrolytic scission to be effective. The separation is limited in a gel because the diffusing ends are ultimately attached to the network. Such ends can be considered as somewhat like geminate pairs and would be subject to more rapid recombination than in uncrosslinked elastomers. This would reduce $\left[\mathrm{A}_{e}\right]$ by increasing the rate of the back reaction. An effect such as this ought to be more significant if the crosslinking density is high.

Hydrophobic gelling agents could reduce [ $\mathrm{A}_{e}$ ] simply by reducing the solubility of water in polymers.

The decreases in $k$ and $k^{\prime}$ as [W] increases are interesting. Esterification has been reviewed quite recently, but little mention was made of hydrolysis. ${ }^{10}$ The esterification process in bulk media at low $[\mathrm{A}]$ and $[\mathrm{H}]$ is usually third order overall, first order in alcohol, and second order in acid. ${ }^{10}$ The mechanism is thought to be dimerization of two acid groups to form an associated ion pair, which reacts slowly with the alcohol. The rate constant $k^{\prime}$ is the product of the equilibrium constant for formation of the ion pair and the rate constant for its slow reaction. Increases in [W] could decrease $k^{\prime}$ by decreasing either of these constants by solvating any of the participating species.

Another review contains much more material on hydrolysis but self-catalyzed hydrolysis is not mentioned. ${ }^{11}$ Frequently the rate of mineral-acid-catalyzed hydrolysis is first order in $\mathrm{H}^{+}$and first order in ester. The order with respect to water is uncertain, with workers finding first order in some systems and second order in others. First or second order in [W] implies that our $k$ should be independent of [W] or should increase with [W]. Instead $k$ decreases as [W] increases. The mechanism of most mineral-acid-catalyzed hydroiyses is written as addition of water to a protonated ester which decomposes to alcohol and carboxylic acid.

In our system the concentration of protons is small so that other intermediates might be involved. Our finding seems to imply that additional water interfers with the formation or decomposition of whatever the intermediates are.

## CONCLUSIONS

Equilibrium acid contents are similar in polyesters of low molecular weight and in polyester-polyurethane elastomers made with the same polyesters. Equilibration does not set a practical limit on hydrolysis in such polyurethanes but might do so in highly crosslinked systems. Reesterification at low relative humidity is too slow to be a useful means of regenerating polyester polyurethanes.

Dr. C. S. Schollenberger of the B. F. Goodrich Company provided PBA 930 and PBA 1090.

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[^1]:    ${ }^{\text {a }}$ Except as indicated: $[\mathrm{W}]=\mathrm{k}^{\prime \prime}[\mathrm{A}]+\left[\mathrm{W}_{0}\right]$ where k " in $\mathrm{mol} \mathrm{H}_{2} \mathrm{O} / \mathrm{mol}$ acid and $\left[\mathrm{W}_{0}\right]$ in $\mathrm{mol} / \mathrm{g}$ are 2.1 and $2.9 \times 10^{-3}$ at $93 \% \mathrm{RH}, 0.55$ and $0.64 \times 10^{-3}$ at $\sim 50 \% \mathrm{RH}$, and 0.17 and $0.25 \times 10^{-3}$ at $25 \%$ RH.
    ${ }^{\mathrm{b}}[\mathrm{W}]=\mathrm{k}^{\prime \prime}[\mathrm{A}]^{2}+\left[\mathrm{W}_{0}\right]$ where $\mathrm{k}^{\prime \prime}=555\left(\mathrm{~mol} \mathrm{H}_{2} \mathrm{O} / \mathrm{mol} \mathrm{acid}\right)^{2}$ and $\left[\mathrm{W}_{0}\right]=2 \times 10^{-3} \mathrm{~mol} / \mathrm{g}$.
    $c[\mathrm{~W}]=\mathrm{k}^{\prime \prime}[\mathrm{A}]^{2}+\left[\mathrm{W}_{0}\right]$ where $\mathrm{k}^{\prime \prime}=614(\mathrm{~mol} \mathrm{H} 2 \mathrm{O} / \mathrm{mol} \text { acid })^{2}$ and $\left[\mathrm{W}_{0}\right]=2.9 \times 10^{-3}$.

